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MODELING RADIANT-HEAT-TRANSFER PROBLEMS IN MEDIA OF NONPLANE GEOMETRY

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Approximate and numerical methods of solution of radiation-transfer equations in cylindrical and spherical media are proposed, the spectroscopic luminescence characteristics of infinite and finite cylinders are analyzed, and an algorithm for their calculation is given.

In many problems of radiant heat transfer, it is necessary to take account of multiple-scattering processes, since the heat carrier in various power stations is a two-phase gas–solid-particle system. The investigation of multiple-scattering laws is also of great importance for other fields of physics and for physical-engineering applications (atmospheric optics, the energetics of planetary atmospheres, the problem of spacecraft entry into the atmospheric layer, the interaction of laser radiation with matter, etc.). The intensification of modern power stations, associated with the considerable increase in heat-carrier temperature, requires as accurate as possible a determination of their thermodynamic characteristics. At the same time, the radiant component in the total energy balance becomes significant, and therefore the correct solution of radiant-heat-transfer problems is a pressing concern. On the one hand, it is necessary to establish the basis of radiation-transfer equations for real physical models and the limits of applicability of this solution; on the other, it is necessary to use reliable spectroscopic characteristics of the media investigated.

The problem of radiation propagation in two-phase media of nonplane geometry is one of the most important in modern radiation-transfer theory. Because of the great mathematical difficulties involved, approximate [1-3] or numerical [4, 5] methods are usually used for the solution of integrodifferential radiation-transfer equations. Note that the development of approximate methods of solution is expedient both for rapid estimates of the energy characteristics of two-phase nonplane media and for the determination of the best initial approximation in numerical calculations of radiation-transfer equations by iterational methods. The wide use of computers allows numerical experiments to be carried out for diverse physical phenomena, which, in economic terms, is considerably preferable to full-scale experiments and physical modeling. By constructing mathematical models, it is possible to study the important physical laws governing phenomena or to investigate directly conditions of power-

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station operation which cannot be determined either in experimental or in full-scale conditions.

In the present work, approximate analytic expressions are given for the radiative capacity of cylindrical and spherical two-phase media. On this basis, an effective iterative method is developed for the solution of the equations of radiation transfer in two-phase cylindrical (infinite or finite) media. By means of accurate numerical calculations, limits of applicability are established for the approximate relations for the calculation of the luminescence characteristics of an infinite two-phase cylinder.

In essence, the approximate methods of calculating the radiative capacity of plane, cylindrical, and spherical two-phase homogeneous media proposed in [3, 6, 7] involve the preliminary approximate determination of the source functions, followed by direct integration of the radiation-transfer equations. Suppose that the two-phase medium is characterized by an attenuation coefficient $\alpha = \varkappa + \sigma$, and its size by the quantity L in the case of a plane layer or R in the case of a cylindrical or spherical medium. The indicatrix of radiation scattering on an elementary volume is taken to be spherical. The following representation of the indicatrix may be used to take account of its nonspherical form in considering multiple scattering processes [3, 5, 6]:

$$p(\mathbf{l}, \mathbf{l}') = a + 4\pi(1-a)\,\delta(\mathbf{l}-\mathbf{l}'). \tag{1}$$

This representation reduces the initial equation to the case of isotropic scattering with the scattering coefficient σ replaced by the quantity $a\sigma$. The parameter a denotes twice the hemispherical fraction of forward scattering in the interaction of radiation with an elementary volume of the medium: $a = 2\beta$.

The radiation-transfer equation in a two-phase homogeneous medium may be written in the form [8]

$$(\mathbf{l}\nabla)_i I_i(t, \tau, \mathbf{l}) + I_i(t, \tau, \mathbf{l}) = S_i(t, \tau, \mathbf{l}). \tag{2}$$

Here the subscript i = 1, 2, 3 denotes the case of plane, cylindrical (infinite), and spherical media, respectively

$$I_1 = I_1(t, \mu),\ S_1(t) = \frac{\lambda}{2}\int_{-1}^{1} I_1(t, \mu)\,d\mu + S_0(t), \quad (\mathbf{l}\nabla)_1 = \mu\frac{d}{dt}; \tag{3}$$

$$I_2 = I_2(\tau, \theta, \varphi),\ S_2(\tau) = \frac{\lambda}{4\pi}\int_0^{2\pi} d\varphi \int_0^{\pi} I_2(\tau, \theta, \varphi)\sin\theta\,d\theta + S_0(\tau), \quad (\mathbf{l}\nabla)_2 = \sin\theta\cos\varphi\frac{\partial}{\partial\tau} - \frac{\sin\theta\sin\varphi}{\tau}\frac{\partial}{\partial\varphi}; \tag{4}$$

$$I_3 = I_3(\tau, \theta),\ S_3(\tau) = \frac{\lambda}{2}\int_0^{\pi} I_3(\tau, \theta)\sin\theta\,d\theta + S_0(\tau), \quad (\mathbf{l}\nabla)_3 = \cos\theta\frac{\partial}{\partial\tau} - \frac{\sin\theta}{\tau}\frac{\partial}{\partial\theta}. \tag{5}$$

Under local thermodynamic equilibrium, the function $S_0(t)$ is

$$S_0(t) = (1-\lambda)\,B(T). \tag{6}$$

In the case where no radiation falls on the medium from without, the boundary conditions for the equation take the form

$$I_1(0, \mu)|_{\mu>0} = I_1(t_0, \mu)|_{\mu<0} = 0, \quad I_2(\tau_0, \mathbf{l})|_{(\mathbf{l}\mathbf{n})<0} = I_3(\tau_0, \theta)|_{\pi\leqslant\theta\leqslant2\pi} = 0. \tag{7}$$

The solution of the problem in Eqs. (2), (3), and (7) for a plane layer is given in [6]. The following expressions are obtained for the hemispherical radiation intensities of a plane layer

$$\frac{1}{B} I_1^+(t) = \frac{1}{B}\int_0^1 I_1(t, \mu)\, d\mu = 1 - A_1 e^{-k_1 t} - A_2 R e^{-k_1(t_0 - t)}, \tag{8}$$

$$\frac{1}{B} I_1^-(t) = \frac{1}{B}\int_{-1}^0 I_1(t, \mu)\, d\mu = 1 - A_1 R e^{-k_1 t} - A_2 e^{-k_1(t_0 - t)},$$

$$B = \frac{\varkappa_g B(T_g) + \varkappa_{so} B(T_{so})}{\varkappa_g + \varkappa_{so}}, \tag{9}$$

$$aA_1 = 1 - E_1 - R(1 - E_2)e^{-k_1 t_0},\ aA_2 = 1 - E_2 - R(1 - E_1)e^{-k_1 t_0},$$

$$a = 1 - R^2 e^{-2k_1 t_0},\ R = \frac{\delta - 1}{\delta + 1},\ k_1 = 2\delta(1 - \lambda),\ \delta = \left(1 + \frac{2\beta\lambda}{1 - \lambda}\right)^{1/2}. \tag{10}$$

The functions $I_{0k} = E_k B$ ($k = 1, 2$) are the radiation intensities falling from outside on the layer from the left ($\mu > 0$) and from the right ($\mu < 0$), while the subscripts g and so denote the gas phase and the solid phase, respectively. When $E_1 = E_2 = 0$, the well-known relation for the hemispherical emissivity of a two-phase layer is obtained [9, 10]:

$$\varepsilon_1^p(t_0) = \frac{I_1(t_0)}{B} = \frac{I_2(0)}{B} = (1 - R)\,\frac{1 - e^{-k_1 t_0}}{1 + Re^{-k_1 t_0}}, \tag{11}$$

where R is the coefficient of diffuse reflection of an infinitely large optical thickness.

Using Eq. (8), an approximate relation for the source function $S_1(t)$ may be formulated:

$$S_1(t) = \frac{\lambda}{2}\int_{-1}^1 I_1(t, \mu)\, d\mu + S_0(t) = \frac{\lambda}{2}\left[I_1^+(t) + I_1^-(t) \right] + (1 - \lambda)B. \tag{12}$$

Then, substituting $S_1(t)$ into the initial equation, the directed radiative capacity of a two-phase layer may be found [6]:

$$\varepsilon(t, \mu) = \frac{I_1(t, \mu)}{B} = 1 - (1 - E_1)e^{-\frac{nt}{\mu}} - \lambda\beta(1 + R)\left[\frac{e^{-k_1 t} - e^{-\frac{nt}{\mu}}}{n - \mu k_1} A_1 + \frac{e^{k_1 t} - e^{-\frac{nt}{\mu}}}{n + \mu k_1} A_2 e^{-k_1 t_0}\right] \text{when } \mu > 0,$$

$$\varepsilon(t, \mu) = 1 - (1 - E_2)e^{\frac{n(t_0 - t)}{\mu}} - \lambda\beta(1 + R)\left[\frac{e^{-k_1 t} - e^{\frac{n(t_0 - t)}{\mu} - k_1 t_0}}{n - \mu k_1} A_1 + \frac{e^{-k_1(t_0 - t)} - e^{\frac{n(t_0 - t)}{\mu}}}{n + \mu k_1} A_2\right] \text{when } \mu < 0, \tag{13}$$

where

$$n = 1 - \lambda + 2\beta\lambda.$$

Using Eqs. (8) and (13), the hemispherical and directed emissivity of a plane two-phase medium may be analyzed as a function of the optical characteristics of the medium, the boundary conditions, and the conditions of observation [6]. The results of these calculations have been used to construct nomograms expedient for the analysis of the layer emissivity.

In [11], the emissivity of an inhomogeneous two-phase layer bounded by reflecting and radiating surfaces was analyzed. As an example, curves of the radiation emitted from a two-

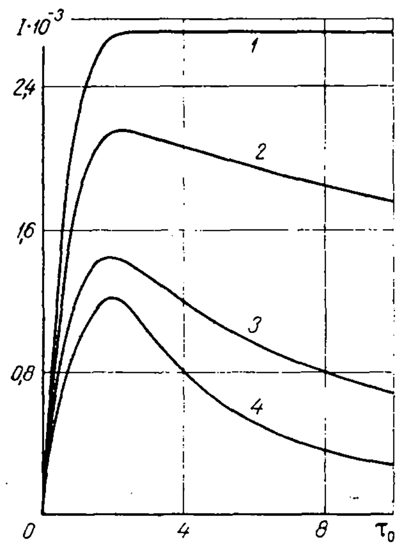

Fig. 1. Luminescence intensity of nonisothermal layer as a function of the optical thickness for different temperature gradients (W/μm•sr): 1) c = 0; 2) 0.2; 3) 0.6; 4) 1.0.

phase layer with transparent boundary surfaces are shown in Fig. 1. The corresponding temperature gradient is given by the relation

$$T(t) = T_0 \exp[-c(t-t_0)/2]^2. \tag{14}$$

Using the Eddington approximation to solve Eqs. (2), (4), and (5) [12-14], the function $J(\tau)$ for cylindrical and spherical media may be determined:

$$J_2(\tau) = B[1 - A\tilde{I}_0(k\tau_0)], \tag{15}$$

where

$$A^{-1} = \tilde{I}_0(k\tau_0) + \frac{2}{3} k\tilde{I}_1(k\tau_0), \quad k = \sqrt{3(1-\lambda)},$$

and

$$J_3(\tau) = B\left(1 - C\frac{\tau_0 shk\tau}{\tau shk\tau_0}\right), \tag{16}$$

$$C^{-1} = \frac{1}{\tau_0}\left(\tau_0 - \frac{2}{3} + \frac{2}{3} k\tau_0 \frac{1+e^{-2k\tau_0}}{1-e^{-2k\tau_0}}\right).$$

Substituting Eq. (15) into the corresponding initial transfer equations, the radiation intensity may be found in the case of cylindrical and spherical configurations of a two-phase medium [3]:

$$\frac{1}{B} I_2(\tau, \theta, \varphi) = 1 - \exp\left(-\frac{\sqrt{\tau_0^2 - \tau^2\sin^2\varphi} + \tau\cos\varphi}{\sin\theta}\right) - $$

$$- \lambda A \exp\left(-\frac{\tau\cos\varphi}{\sin\theta}\right) \int\limits_{-\sqrt{\tau_0^2 - \tau^2\sin^2\varphi}}^{\tau\cos\varphi} \tilde{I}_0\left(k\sqrt{x^2 - \tau^2\sin^2\varphi}\right) \exp\left(\frac{x}{\sin\theta}\right) \frac{dx}{\sin\theta}, \tag{17}$$

$$\frac{1}{B} I_3(\tau, \mu) = 1 - \exp\left(-\tau\mu - \sqrt{\tau_0^2 - \tau^2 + \tau^2\mu^2}\right) - $$

$$-\frac{\lambda\tau_0}{\mathrm{sh}k\tau_0}C\int\limits_{-\sqrt{\tau_0^2-\tau^2+\tau^2\mu^2}}^{\tau\mu}\frac{\mathrm{sh}\,(k\sqrt{x^2+\tau^2-\tau^2\mu^2})}{\sqrt{x^2+\tau^2-\tau^2\mu^2}}\exp(x-\tau\mu)\,dx. \tag{18}$$

Using certain physical and mathematical simplifications, the basis of which is given in [3], relations for the emissivity of a cylinder and a sphere may be written in a form convenient for practical use

$$\varepsilon_2(\tau_0,\ \theta)=1-e^{-\frac{\tau_0}{\sin\theta}}-\frac{6\lambda\left(2-\frac{k\tau_0\delta}{3k\tau_0+\delta^2}\right)\left(1-e^{-\frac{k\tau_0}{\delta}}\right)}{(4+k\sin\theta)[3+2k(1-e^{-\frac{k}{2}\tau_0})]}, \tag{19}$$

$$\delta=4k\sin\theta/(4+k\sin\theta),$$

$$\varepsilon_2^{\mathrm{p}}(\tau_0)=1-e^{-2\tau_0}-\frac{24\lambda\left[1-\frac{2k\tau_0(8+k)}{16k+3\tau_0(8+k)^2}\right][1-e^{-\frac{\tau_0}{4}(8+k)}]}{(8+k)[3+2k(1-e^{-\frac{k}{2}\tau_0})]}, \tag{20}$$

$$\varepsilon_3(\tau_0,\ \mu)=(1-e^{-2\tau_0\mu})\left[1-\frac{\lambda(3k+\tau_0)C}{\tau_0+3k+\mu k\tau_0}\right], \tag{21}$$

$$\varepsilon_3^{\mathrm{p}}(\tau_0)=(1-e^{-\tau_0})\left[1-\frac{2\lambda(\tau_0+3k)C}{6k+\tau_0(2+k)}\right]. \tag{22}$$

A detailed analysis of Eqs. (19)-(22) is given in [3]. Comparison of these equations with the corresponding expressions for a plane layer shows that the use of the "plane-layer approximation" may lead to considerable errors when $\tau_0 < 5$. Note that for purely radiating media

$$\varepsilon_1^{\mathrm{p}}(\tau_0):\varepsilon_2^{\mathrm{p}}(\tau_0):\varepsilon_3^{\mathrm{p}}(\tau_0)=(1-e^{-4\tau_0}):(1-e^{-2\tau_0}):(1-e^{-\tau_0}), \tag{23}$$

which is in good agreement with the data of [15]. Nomograms for the calculation of the emissivity of a two-phase cylinder in accordance with Eq. (19) are shown in Fig. 2.

To allow accurate calculations of the emissivity of an infinite two-phase medium to be made, and to establish the basis for limits of applicability of Eqs. (19) and (20), a numerical method of integration of Eq. (2) has been developed; the method is described in detail in [7]. The Vladimirov method [16] and the discrete-ordinate method are used to write an algorithm; the best first iteration is found to be Eq. (15). The grid for the solution of Eq. (2) according to the method developed is shown in Fig. 3a. At the grid points, the solution is found from the following relations:

$$I_{k,i,j}=I_{k,i-1,j}\cdot q_{k,i,j}+\sqrt{1-\gamma_k^2}\,S_{k,i-1}p_{k,i,j}+(1-q_{k,i,j}-p_{k,i,j})\sqrt{1-\gamma_k^2}\,S_{k,i}, \tag{24}$$

$$S_{k,i}=\frac{1}{\sqrt{1-\gamma_k^2}}\left[\frac{\lambda}{\pi}I_0(\tau_i)+(1-\lambda)B(\tau_i)\right], \tag{25}$$

$$q_{k,i,j}=\exp\left(-\frac{\Delta x_{ij}}{\sqrt{1-\gamma_k^2}}\right),\quad p_{k,i,j}=\frac{\sqrt{1-\gamma_k^2}}{\Delta x_{ij}}(1-q_{k,i,j}),$$

$$I_0(\tau_i)=\sum_k A_k\sum_n[B_nI(\tau_i,\mu_n,\gamma_k)+C_{n-1}I(\tau_i,\ \mu_{n-1},\ \gamma_k)], \tag{26}$$

where

$$B_n=-\frac{1}{\Delta x_n}\left[\Delta y_n+x_{n-1}\left(\arcsin\frac{x_n}{\tau_i}-\arcsin\frac{x_{n-1}}{\tau}\right)\right],$$

$$C_{n-1}=\frac{1}{\Delta x_n}\left[\Delta y_n+x_n\left(\arcsin\frac{x_n}{\tau_i}-\arcsin\frac{x_{n-1}}{\tau_i}\right)\right],$$

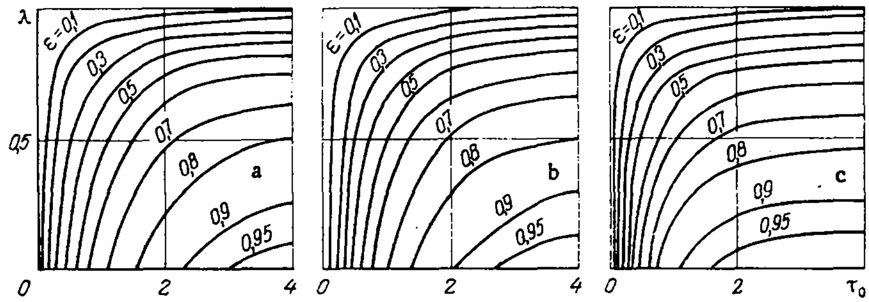

Fig. 2. Nomograms for the calculation of the emissivity of a two-phase cylindrical medium for θ = 90° (a), θ = 60° (b), θ = 30° (c).

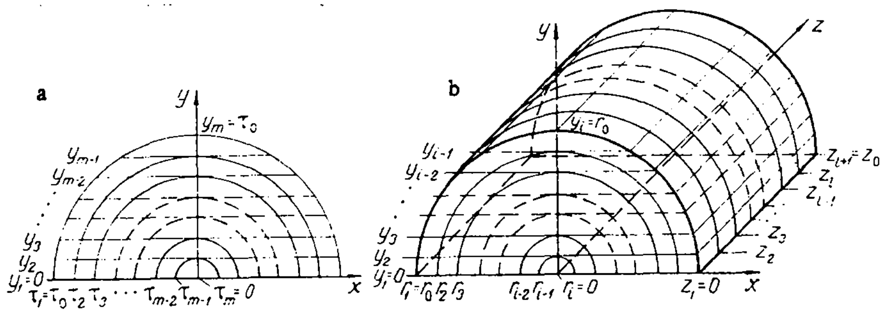

Fig. 3. Variable-specification grid for the calculation of the radiation of an infinite (a) and finite (b) two-phase cylinder.

$$\Delta x_n = x_n - x_{n-1},\ \Delta y_n = y_n - y_{n-1},$$

$$I_k(-\sqrt{\tau_0^2 - y^2},\ y) = 0 \text{ when } 0 \leqslant y \leqslant \tau_0,\ k = 1,\ 2,\ \ldots,\ n. \tag{27}$$

Here $I_{k,i,j} \equiv I(x_{ij}, y_j, \gamma_k)$; x_{ij} is the value of the coordinate x at the intersection of the straight line y_j = const and the arc τ_i = const; A_k and γ_k are the positive weights and nodes in the Gauss quadrature formula for γ on the segment [0, +1]; $x = \tau\mu$, $y = \tau\sqrt{1-\mu^2}$.

The following scheme is used for the calculation. Equation (15) is used to calculate the values of $J(\tau_i)$, i = 1, 2, ..., m. Since $I_0(\tau) = \pi J(\tau)$, the values of $I_0(\tau)$ obtained are substituted in Eqs. (25). Then the values of $S_k(\tau_i)$ are substituted in Eq. (24) and the values of $I_{k,i,j}$ are found at all the grid points. Substituting these values into Eq. (26), $I_0(\tau_i)$ is found in a new approximation; summation over n in Eq. (26) is taken over the grid points on the arcs τ_i = const (Fig. 3a). This process is repeated until the accuracy specified in advance is attained.

Numerical calculations on EC-1030 and EC-1022 computers have demonstrated the efficiency of this method of solution. For large values of the survival probability of the quantum ($\lambda \sim 0.999$) and the optical thickness ($\tau_0 \sim 15$), the calculation time does not exceed a minute. On the average, the calculation of a single variant takes ~10-15 sec. A calculation accuracy of 0.1% is reached after 2-3 iterations for $\tau_0 \sim 0.01$-0.1 and after ≤10 iterations for $\tau_0 \sim 1$; with further increase in τ_0 the number of iterations increases (for $\tau_0 \sim 15$, ~100 iterations are required).

Comparison of the results obtained with calculations by Eq. (19) reveals good agreement when $\lambda \leq 0.90$-0.95. With increase in optical radius of the cylinder, the error of the approximate calculation using Eq. (19) rapidly falls. For $\tau_0 \geq 0.1$ and $\lambda \leq 0.9$, the error practically never exceeds 20-25%. Note that accurate values of the emissivity are used in constructing nomograms (see Fig. 2) for small optical thicknesses and large values of the quantum-survival probability. As established in [3], the luminescence indicatrix of a cylindrical two-phase medium changes greatly on passing from small to large optical thicknesses, especially when λ is close to unity. For example, the degree of anisotropy of the luminescence indicatrix $I(\tau_0, \pi/2)/I(\tau_0, 0)$ is ~10 for $\tau_0 = 0.05$ and $\lambda = 0.999$, 0.8 for $\tau_0 = 1.0$, and only 0.56 for $\tau_0 = 10$. Table 1 gives accurate values of the hemispherical emissivity of an infinite cylindrical two-phase medium for different τ_0 and λ.

In modeling thermal-engineering problems, it is very important from a practical viewpoint to establish the possibility of introducing certain effective values that determine the emissivity of the medium under investigation. In the case of a radiating plane layer, the arithmetic mean T_A and geometric mean T_G temperatures usually used in practice [17, 18] must be replaced by the following effective tempeature [19]:

$$T_{ef}=\frac{hc}{\lambda k}\left[\ln\left(1+\frac{1}{A}\right)\right]^{-1}, \tag{28}$$

where

$$A=\frac{2}{1-\exp(-2t_0)}\int\limits_0^{t_0}\frac{\exp[-2(t_0-x)]}{\exp\left[\frac{hc}{\lambda kT(x)}\right]-1}\,dx. \tag{29}$$

A detailed analysis of Eq. (28) as a function of the temperature profile and optical characteristics of the medium is given in [20]. The use of T_A and T_G in practical calculations may lead to large errors (Fig. 4). Figure 4 corresponds to a Schlichting profile [21]

$$T(t)=T_{\min}+(T_{\max}-T_{\min})\left\{1-\left|1-\frac{2t}{t_0}\right|^{\frac{3}{2}}\right\}^{1,6}.$$

In the case of a cylindrical two-phase medium, the introduction of effective values of the temperature or the absorption coefficient, given by the usually adopted methods of averaging, leads to large errors [7]. With increase in optical thickness, the error of the calculations rapidly increases. This behavior was established in [7] for the following dependences of the temperature $T(\tau)$ and the absorption coefficient $\varkappa(\tau)$ on the optical thickness of the cylinder:

$$T(\tau)=T_0\exp(-\alpha\tau^2),\quad \varkappa(\tau)=\varkappa_0\exp(-\alpha\tau^2),$$

where the constant α, determining the gradient of the change in $T(\tau)$ and $\varkappa(\tau)$, varies over a wide range.

To investigate the features of radiation propagation in a finite cylindrical medium, the following transfer equation must be solved [8]:

$$\cos\theta\frac{\partial I}{\partial z}+\sin\theta\cos\varphi\frac{\partial I}{\partial r}-\frac{\sin\theta\sin\varphi}{r}\frac{\partial I}{\partial\varphi}+[\varkappa(r,z)+\sigma(r,z)]I$$
$$=\frac{\sigma(r,z)}{4\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\pi}I(r,z,\theta,\varphi)\sin\theta\,d\theta+\varkappa(r,z)B[T(r,z)]. \tag{30}$$

If radiation is incident on the end surfaces of the cylinder, the boundary conditions for Eq. (30) take the form

$$\text{a) } I(R,z,\theta,\varphi)|_{(\mathbf{l}\mathbf{n})<0}=0,$$
$$\text{b) } I(r,0,\theta,\varphi)|_{0\leqslant\theta\leqslant\frac{\pi}{2}}=I_1,\quad I(r,z_0,\theta,\varphi)|_{\frac{\pi}{2}\leqslant\theta\leqslant\pi}=I_2. \tag{31}$$

Using the new variables $\gamma = \cos\theta$, $\mu = \cos\varphi$, $x = r\mu$, and $y = r\sqrt{1-\mu^2}$, an algorithm analogous to Eqs. (24)-(26) may be constructed. In contrast to Eqs. (24)-(26), geometric (and not optical) coordinates are used in the algorithm, as it is necessary to take account of the inhomogeneity of the medium in both r and z, and also the absence of symmetry in θ is taken into account, which leads to consideration of the interval [−1, 1] in γ, and not [0, 1]. The variable-specification grid for the solution of the initial system of equations is shown in Fig. 3b.

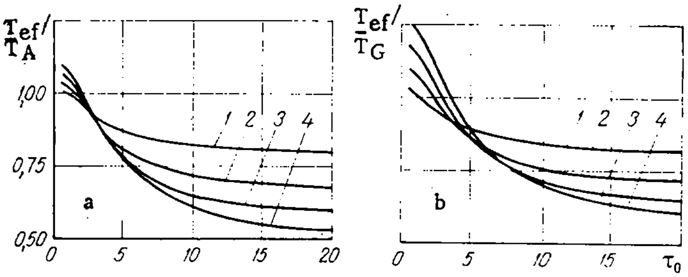

Fig. 4. Ratio of T_{ef} to $\overline{T}_A$ (a) and $\overline{T}_G$ (b) as a function of the optical thickness of the layer ($\lambda T_{min} = 5 \cdot 10^{-3}$); 1) T_{max}/T_{min} = 1.5; 2) 2.0; 3) 2.5; 4) 3.0.

In this case the solution of Eq. (30) with the boundary conditions in Eq. (31) at all the grid points is found from the expression

$$I_{k,i,j,l} \equiv I_k(x_{ij},\ y_j, z_l) = \frac{\dfrac{\sqrt{1-\gamma_k^2}}{\Delta x_{ij}} I_{k,i-1,j,l} + \dfrac{\gamma_k}{\Delta z_l} I_{k,i,j,l-1} + S_{il}}{\dfrac{\sqrt{1-\gamma_k^2}}{\Delta x_{ij}} + \dfrac{\gamma_k}{\Delta z_l} + \varkappa(r_i,\ z_l) + \sigma(r_i,\ z_l)}, \tag{32}$$

where

$$S_{i,l} = \frac{\sigma(r_i\ z_l)}{2\pi} I_0(r_i,\ z_l) + \varkappa(r_i,\ z_l) B(r_i,\ z_l), \tag{33}$$

and $I_0(r_i,\ z_l)$ is determined from Eq. (26) for each value of $z = z_l$. The iterational process reduces to the following. A certain approximate expression for $I(r,\ z,\ \gamma,\ \mu)$ is specified, and the value of $I_0(r,\ z)$ and hence $S(r,\ z)$ is calculated for all values of r_i and z_l. Then, using the boundary conditions in Eqs. (31a) and (31b), the solution for the plane $z = \Delta z_1$ is found from Eq. (32) for all the values $k = 1, 2, \ldots, n$ $(0 \leqslant \theta \leqslant \pi/2)$. This procedure is repeated for each cross section z_l = const. Analogously, the solutions for $k = n+1, n+2, \ldots, 2n$ $(\pi/2 \leqslant \theta \leqslant \pi)$ is found for the cross section $z = z_0 - \Delta z_l$ = const (l is the number of divisions along the z axis), and then the solution is extended over the whole cross section z_l = const for these values of k (and angles θ). Substituting the resulting values of $I_{k,i,j,l}$ in the expression for $I_0(r,\ z)$, the following approximation is obtained. The iterative procedure is continued until the previously specified accuracy of the calculation is attained. The given algorithm is realized as a program on the EC-1030 computer; the program is written in Fortran IV. The first approximation adopted in this program is the expression

$$I_0(r,\ z) = 2\pi\{I_1 e^{-\tau_1(r,z)} + I_2 e^{-\tau_2(r,z)} + J_2(r,\ z)\}, \tag{34}$$

where

$$\tau_1(r,z) = \int_0^z [\varkappa(r,\ z') + \sigma(r,\ z')]\, dz',\quad \tau_2(r,\ z) = \int_z^{z_0} [\varkappa(r,\ z') + \sigma(r,\ z')]\, dz', \tag{35}$$

while $J_2(r,\ z)$, determining the contribution of the intrinsic radiation of the medium, is given, according to Eq. (15), by the relation

$$J_2(r,\ z) = [1 - A\tilde{I}_0(k\tau)]\, B(r,\ z), \tag{36}$$

$$\tau = \tau(r,\ z) = \int_0^r [\varkappa(r',\ z) + \sigma(r',\ z)]\, dr,\quad \tau_0 = \tau(R,\ z),$$

$$\lambda = \frac{1}{\tau_0} \int_0^R \sigma(r',\ z)\, dr'.$$

TABLE 1. Results of Accurate Calculation of the Emissivity of an Infinite Two-Phase Cylinder*

τ_0	λ							
	0,3	0,7	0,85	0,9	0,95	0,99	0,995	0,999
0,01	0,134—1(2)	0,575—2(2)	0,288—2(2)	0,192—2(2)	0,961—3(2)	0,192—3(2)	0,961—4(2)	0,192—4(2)
0,05	0,634—1(2)	0,278—1(2)	0,140—1(2)	0,939—2(2)	0,471—2(2)	0,944—3(3)	0,472—3(3)	0,945—4(3)
0,1	0,120(2)	0,538—1(3)	0,273—1(3)	0,183—1(3)	0,921—2(3)	0,185—2(3)	0,926—3(3)	0,185—3(3)
0,5	0,421(3)	0,219(4)	0,119(5)	0,820—1(5)	0,423—1(6)	0,868—2(6)	0,436—2(6)	0,879—3(6)
1,0	0,610(3)	0,360(5)	0,211(7)	0,149(7)	0,796—1(8)	0,168—1(8)	0,847—2(8)	0,171—2(9)
5,0	0,851(3)	0,630(6)	0,465(9)	0,378(11)	0,250(15)	0,703—1(23)	0,372—1(25)	0,782—2(26)
10,0	0,877(3)	0,664(6)	0,507(10)	0,424(14)	0,301(24)	0,104(59)	0,600—1(71)	0,133—1(85)

*The notation 0,192-3 denotes $0.192 \cdot 10^{-3}$; the figure in brackets is the number of iterations.

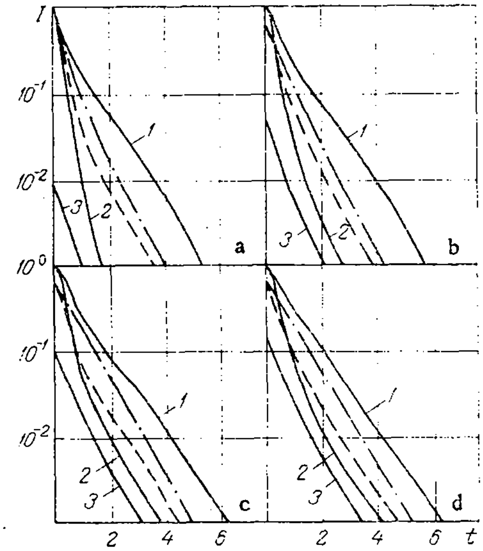

Fig. 5. Radiation from a two-phase finite cylinder with B = 0, I_1 = 1, I_2 = 0: a) λ = 0.1; b) 0.5; c) 0.9; d) 1.0. 1) θ = 158°; 2) 21°; 3) 76°.

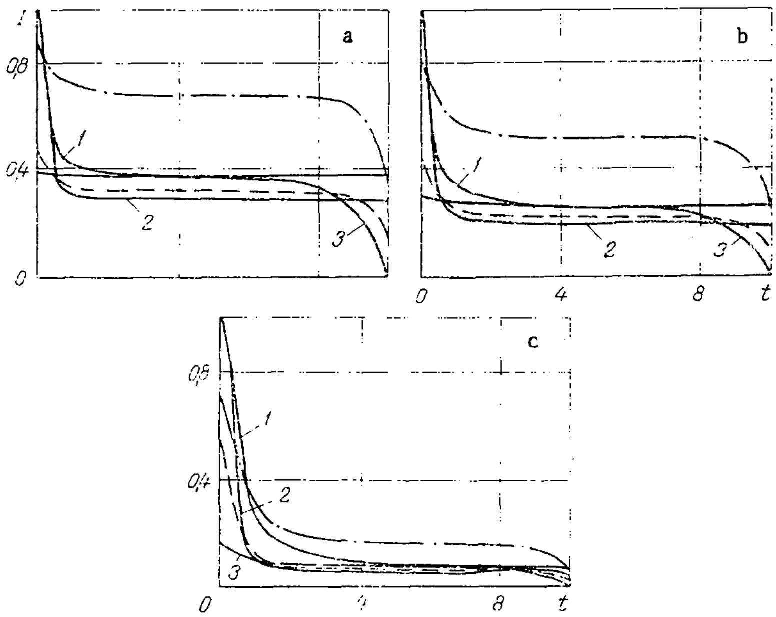

Fig. 6. Radiation from two-phase finite cylinder with B = 1, I_1 = 1, I_2 = 0: a) λ = 0.1; b) 0.5; c) 0.9. 1) θ = 158°; 2) 21°; 3) 76°.

Preliminary calculations show that of all the trial functions Eq. (34) provides the most rapid convergence of the iterative process. As an example, some results obtained for the radiation from a two-phase finite cylinder are shown in Figs. 5 and 6, where the continuous curves show the intensity of the emitted radiation averaged over φ : $(2/\pi)\int_0^{\frac{\pi}{2}} I(R, z, \theta, \varphi)d\varphi$; the dashed curves show the mean radiation intensity over the side surface of the cylinder: $(1/4\pi)\int_{(4\pi)} I(R, z, \theta, \varphi)d\Omega$; the dash-dot curves show the mean radiation intensity over the cylinder axis. Figures 5 and 6 correspond to a cylinder of the same size (τ_0 = 1, t_0 = 10) but with different values of the internal-source function (B = 0 in Fig. 5 and B = 1 in Fig. 6).

NOTATION

$I_\nu(r, z, \mathbf{l})$, intensity of radiation of frequency ν at the point (r, z) and in the direction $\mathbf{l} = \mathbf{l}(\theta, \varphi)$; $B\nu(T) = (2h\nu^3/c^2)(e^{h\nu/kT} - 1)^{-1}$, intensity of Planck radiation; $J_\nu(r, z) = (1/4\pi)\int_{(4\pi)} I_\nu(r, z, \mathbf{l})d\Omega$, mean radiation intensity; $S(r, z)$, radiation-source function; $S_0(r, z)$, internal-radiation-source function; $\varkappa$, σ, absorption and scattering coefficients; $\alpha = \varkappa + \sigma$, attenuation coefficient of medium; $\lambda = \sigma/\alpha$, quantum-survival probability; $p = p(\mathbf{l}, \mathbf{l}')$, radiation-scattering indicatrix on an elementary volume; β, hemispherical fraction of forward scattering for radiation scattering with elementary volume of the medium; I_1, I_2, radiation intensity falling on the ends of a two-phase cylinder from left and right, respectively; $T_0 = T_0(z)$, temperature along cylinder axis; $T_S = T_S(z)$, surface temperature of cylinder; $0 \le z \le L$, cylinder length (or plane-layer thickness); $0 \leqslant r \leqslant R$, cylinder (or sphere) radius; $0 \leqslant \tau = \int_0^r \alpha dr \le \tau_0 = \int_0^R \alpha dr$, optical thickness of cylinder along radius; $0 \leqslant t = \int_0^z \alpha dz \leqslant t_0 = \int_0^L \alpha dz$, optical length of cylinder; $\tilde{I}_n(x)$, n-th order Bessel function with imaginary argument; $\gamma = \cos\theta$; $\mu = \cos\varphi$; $\mathbf{n}$, external normal to boundary surface of medium.

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